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AUTHOR Beattie, Ian D.; Deichmann, John W.
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ABSTRACT

The purpose of the study was to investigate the effects of the presentation mode (vertical, horizontal) and the placeholder position (left, middle, right) of the frames of workbooks upon first and second grade children's ability to solve number sentences as measured by the rate and type of error. Effects over addition and subtraction were also investigated. The workbooks of 132 children in Southern Illinois, each representing an entire year's written work, were studied. No statistical analysis was applied to the data, but results indicated that the variables did produce differential effects over grade levels both in the rate and type of error produced, that certain number sentence formats appeared to present more difficulty than others, and that the amount of practice was an important factor. (JM)

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Error Trends in Solving Number Sentences in Relation to Workbook Format Across 1st and 2nd Grades

Ian D. Beattie
Southern Illinois University

John W. Deichmann
Southern Illinois University

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Abstract

The effects of varying the presentation mode and placeholder position upon the ability of first and second grade children to solve number sentences were measured by rate and type of error produced. The results appear to demonstrate that these variables produce differential effects over grade levels both in the rate and type of error produced. The results also indicate that certain number sentence formats appear to present more difficulty than others and that the amount of practice is an important factor.

Rationale:

The invention and development of the technique of using frames as variables has had a profound effect on elementary school mathematics workbooks. Since frames were first used by UICSM in the 1950's they have been incorporated into elementary workbooks to the extent that a major portion of the practice exercises in mathematics workbooks at the primary level is devoted to the solution of number sentences. Typically these number sentences are presented in a variety of ways with two major variables being presentation mode (horizontal, vertical), and frame or placeholder position (left, middle, right). The effects of these variables upon student ability to solve number sentences or upon the type of error porduced has not been investigated, indicating that the rationale for varying such factors is not empirically based. If mathematics workbooks are to achieve optimum effectiveness, there is great need for data indicating whether different forms of presentation have differential effects on student performance.

The purpose of this study, therefore, was to investigate the effects of presentation mode (vertical, horizontal) and placeholder position (left, middle, right) upon first and second grade children's ability to solve number sentences as measured by rate and type of error. The effects of those variables over operations (addition, subtraction) were also investigated. It was hoped that the normative data thus obtained would generate hypotheses which could then be tested experimentally.

METHOD

Sample

The workbooks of 132 first and second grade children in two Southern Illinois school systems were analyzed. Inclusion in the study was determined by availability, but a wide ability range was represented. These workbooks contained the entire year's written work of those children in mathematics, involving a total of over 17,000 pages of work and 90,000 individual responses.

Classification of sentences

Two presentation modes (H,V) by 3 placeholder (L,M,R) positions yielded 6 distinct types of number sentences for each operation (+,-) producing 12 sentence types in all.

Classification of errors

Errors were classified as being one of 3 types, Computation (C), Process (P), or Random (R). Bases for classification were as illustrated by the following example. In the number sentence 7 + □ = 10, an answer of 17 indicated that the child added 10 and 7, a P error since the answer is in fact obtained by subtraction if the specific number fact is not recalled. Responses such as 16 or 18 were also considered P errors. Responses such as 2, 4, or 1 were considered C errors, since the correct process was apparently used, but the response incorrectly recalled or computed. Responses which could not be placed in either of these two categories were considered R errors, even though in some instances errors were produced in some other systematic way.

PROCEDURE

All analysis was done by two researchers. A preliminary analysis of the workbooks was made and guidelines for classification of categories and errors as previously described were developed. As the work was completed by the children it was graded and errors tabulated according to type of number sentence and type of error by one of the researchers. Doubtful instances were discussed before final classification. The data were organized and tabulated, but no statistical analyses were made.

RESULTS AND DISCUSSION

Table 1 illustrates one unexpected outcome of the study, the disproportionate

representation of the differing types of number sentences. To determine whether this was a characteristic unique to the workbook examined six other workbook series were examined for type of number sentence only. The results were similar to those presented below, indicating that any consideration of rate and type of error must be made within the context of the overall distribution of number sentence type. It can be seen that the bulk of the 1st grade work (80%) is presented horizontally, while the 2nd grade work is fairly evenly distributed (61% of addition sentences are horizontal, 52% of subtraction are horizontal). Also, those sentences which are not presented horizontally are predominantly of the form VR, in fact over 90% at the second grade level. The format VL is not used at all in second grade, while others occur in very limited numbers.

Table 1. Frequency and Percentage Occurrence of Number Sentence Types by Grade Level

	□+y=z	x+□=z	x+y=□	$\frac{□}{+y}{z}$	$\frac{x}{+□}{z}$	$\frac{x}{y}{□}$	Total
	HL	HM	HR	VL	VM	VR	
1	6 1.6	122 32.6	180 48.1	4 1.1	16 4.3	46 12.3	374
2	24 7.4	83 25.5	92 28.2	0 0	10 3.0	117 35.9	326
1	1 .3	71 25.3	159 56.6	3 1.1	11 3.9	36 12.8	281
2	4 1.2	46 14.2	118 36.5	0 0	10 3.1	145 44.9	323
	□-y=x	x-□=z	x-y=□	$\frac{□}{-y}{z}$	$\frac{x}{-□}{z}$	$\frac{x}{y}{z}$ $\frac{x}{z}$ $\frac{-y}{□}$	
	HL	HM	HR	VL	VM	VR	

Error Rate

Percentage error rates relative to V and H formats are presented in Table 2 by operation and grade level.

Table 2. Comparison of Error Rate for Horizontal and Vertical Presentation Formats

	+		-	
V	10.2	5.8	10.8	5.64
H	13.2	9.8	12.3	7.3
	1	2	1	2

It appears that the H mode produces a consistently higher error rate over operations and grade levels.

Percentage error rates relative to placeholder placement are presented in Table 3 by operations and grade level.

Table 3. Comparison of Error Rate for Left, Middle, and Right Placeholder Placement

	+		-	
L	26.2	8.0	25.1	22.6
M	20.0	5.3	13.1	6.2
R	7.6	6.9	12.0	6.2
	1	2	1	2

Clearly, the position of the placeholder produces differential effects over grade levels and operations. Having the placeholder in the left position apparently has no effect on second graders' ability to solve addition number sentences, but does produce a markedly higher error rate for second graders in subtraction and for both operations in first grade. First graders also apparently have trouble in addition when the placeholder is in the middle. Table 4 presents a more detailed breakdown of the error rate patterns.

Table 4. Comparison of Error Rate by Grade Level, Operation, Placeholder Placement and Presentation Mode.

		HL	HM	HR	VL	VM	VR
+	1	31.0	19.55	8.30	18.55	23.83	4.84
	2	8.04	5.34	7.66	None	3.59	6.01
-	1	14.73	15.52	12.39	28.96	8.09	10.22
	2	22.66	6.68	6.99	None	3.70	5.32

It appears that HR produces a small but consistently higher error rate than VR, and that except for addition in first grade HM produces a higher error rate than VM. This is unexpected, in view of the fact that much more practice (see Table 1) is given in both grades to HM than to VM. This is true also for HR and VR at 1st grade while the balance is relatively even for HR and VR at the 2nd grade level. The error rate for HL and VL is striking, but very little practice is given in these forms (0-1.6%) except for HL (in 2nd grade) addition which had an error rate of 7.3%.

Error Type

The distribution of process, computation or random errors in relation to horizontal and vertical presentation mode is shown in Table 5.

Table 5. Distribution of Error Types for Horizontal and Vertical Presentation Formats

		+		-	
V	C	58.8	69.2	71.8	58.0
	P	37.4	27.5	19.8	36.2
	O	3.8	3.3	8.4	5.8
H	C	73.5	53.5	79.9	60.3
	P	19.4	33.9	12.7	21.7
	O	7.1	13.6	7.4	18.0
		1	2	1	2

While the predominant type of error is one of computation, there are some interesting results regarding process errors. Since the amount of practice given in both grades to addition and subtraction is relatively equal, it appears that certain sentence formats are more likely to cause confusion as to which process ought to be used and that these difficulties vary with the grade level.

Table 6. Distribution of Error Types by Placeholder Placement

		+		-	
L	C	69.7	61.8	69.7	79.4
	P	23.8	36.4	23.6	10.3
	O	6.5	1.8	6.7	10.3
M	C	64.5	63.4	81.8	65.7
	P	31.3	16.7	8.6	11.8
	O	4.2	19.9	9.6	22.5
R	C	82.8	50.5	77.5	56.7
	P	7.3	42.2	15.8	31.8
	O	9.9	7.3	6.7	11.5
		1	2	1	2

Regardless of placeholder position the predominant error is one of computation. Sentences with the placeholder left are much more likely to produce errors at the first grade level for both addition and subtraction. This form occurs only 2% of the time. Having the placeholder in the middle in addition also produces a high rate of process errors in first grade. This type occurs 36% of the time, mostly in the horizontal format. At the second grade level the placeholder left also produces a high rate of process errors for addition, a form which occurs less than 7% of the of the time, all in horizontal format. A surprising result is the high rate of process errors for both addition and subtraction at the second grade level when the placeholder is at the right. These forms occur 64% and 81% of the time. Obviously there is a need for studies which will manipulate presentation mode and amount of practice. Table 7 presents more detailed picture of error type occurrence.

Table 7. Comparison of Error Type by Grade Level, Operation Placeholder Placement and Presentation Mode

			HL	HM	HR	VL	VM	VR
+	1	C	84.2	67.3	81.6	31.3	46.6	88.5
		P	10.5	27.7	7.6	59.4	51.6	6.3
		O	5.3	5.0	10.8	9.3	1.8	5.2
	2	C	61.8	62.0	39.0	0	100	67.3
		P	36.4	17.3	51.2	0	0	29.2
		O	1.8	20.7	9.8	0	0	3.5
-	1	C	100	83.1	78.1	64.0	67.5	74.7
		P	0	8.3	15.1	28.0	12.5	19.7
		O	0	8.6	6.8	8.0	20.0	5.6
	2	C	79.3	68.5	54.8	0	40	82.4
		P	10.3	8.7	27.8	0	40	15.7
		O	10.4	22.8	17.4	0	20	1.9

It is apparent that some number sentence formats are more prone to produce errors of process than others, in particular $\frac{\square}{+y}$, $\frac{x}{z}$ at first grade level and $\frac{+\square}{z}$

$x + y = \square$ at the second grade level. Since $x - y = \square$ also produces a substantial number of process errors at second grade level it may be that performance on one form interferes with performance on the other. This does not appear to be so at the first grade level. Other formats with relatively high rates of process errors

are □ + y = z and x at the second grade level and □ at the first grade level. It
$\underline{+y}$ $\underline{-y}$
z

is of interest to note too, that some forms of number sentences are more prone to produce random responses, x at the first grade level and x + □ = z at the second. It
$\underline{-\square}$
z

appears also that as the number of examples of a given form decreases, the error pattern becomes less stable, for example with x - □ = z at the second grade level (n = 10) and □ - y = z at the first grade level (n = 7).

Discussion. While no statistical anaylsis was applied to the data it is apparent that ability to solve number sentences at the first and second grade levels reflects the interaction of a number of variables. Frame placement and presentation mode produce differential effects over grade levels both in the rate and type of error produced. Because of the disproportionate distribution of number sentence types one must be wary of drawing conclusions as to whether number and type of errors are due to the varied amounts of practice or to some inherent perceptual difficulty in the sentence form. It does appear, however, that sentences which occur in small numbers produce not only a high error rate, but also more random and process errors, indicating the need for research into the optimum number of practice exercises. It also appears that with roughly equivalent amounts of practice exercises, certain number sentence formats appear to present more difficulty than others, indicating a need for research which will manipulate the amount of practice as well as presentation mode.